5. A structure shown in the figure is approximated with one triangular element. Plane strain assumption is used.

(a) Calculate the strain–displacement matrix \([B]\).

(b) When nodal displacements are given by \(\{u_1, v_1, u_2, v_2, u_3, v_3\} = \{0, 0, 2, 0, 0, 1\}\), calculate the element strain vector.

**Solution:**

(a) From nodal coordinates: 
\[
\begin{align*}
x_1 &= 0, & y_1 &= 0, \\
x_2 &= 10, & y_2 &= 10, \\
x_3 &= 0, & y_3 &= 20,
\end{align*}
\]
the following coefficients are calculated:

\[
\begin{align*}
b_1 &= y_2 - y_3 = -10, & b_2 &= y_3 - y_1 = 20, & b_3 &= y_1 - y_2 = -10, \\
c_1 &= x_3 - x_2 = -10, & c_2 &= x_1 - x_3 = 0, & c_3 &= x_2 - x_1 = 10.
\end{align*}
\]

Also area \(A = 20 \times 10 / 2 = 100\). Thus, the strain-displacement matrix becomes

\[
[B] = \frac{1}{2A} \begin{bmatrix} b_1 & 0 & b_2 & 0 & b_3 & 0 \\ 0 & c_1 & 0 & c_2 & 0 & c_3 \\ c_1 & b_1 & c_2 & b_2 & c_3 & b_3 \end{bmatrix} = \frac{1}{200} \begin{bmatrix} -10 & 0 & 20 & 0 & -10 & 0 \\ 0 & -10 & 0 & 0 & 0 & 10 \\ -10 & -10 & 0 & 20 & 10 & -10 \end{bmatrix}
\]

(b) For given nodal displacements, strains can be calculated, as

\[
\{\varepsilon\} = [B]\{d\} = \frac{1}{200} \begin{bmatrix} -10 & 0 & 20 & 0 & -10 & 0 \\ 0 & -10 & 0 & 0 & 0 & 10 \\ -10 & -10 & 0 & 20 & 10 & -10 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 2 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \frac{1}{200} \begin{bmatrix} 40 \\ 10 \\ -10 \end{bmatrix}
\]

Thus, \(\varepsilon_{xx} = 0.2, \varepsilon_{yy} = 0.05\), and \(\gamma_{xy} = -0.05\).