6. Calculate the shape function matrix \([N]\) and strain-displacement matrix \([B]\) of the triangular element shown in the figure

![Diagram of a triangular element with nodes labeled 1, 2, and 3, and coordinates (0,0), (1,0), and (0,1) respectively.]

**Solution:**

Since the element has three nodes, start with

\[
\tilde{\mathbf{u}}(x, y) = \alpha_1 + \alpha_2 x + \alpha_3 y.
\]

By substituting nodal values, we have

\[
\begin{align*}
  u_1 &= \alpha_1 \\
  u_2 &= \alpha_1 + \alpha_2 \\
  u_3 &= \alpha_1 + \alpha_3
\end{align*}
\]

Thus, the approximate solution becomes

\[
\tilde{\mathbf{u}}(x, y) = u_1 + (u_2 - u_1)x + (u_3 - u_1)y
\]

\[
= (1 - x - y)u_1 + xu_2 + yu_3
\]

From the above approximation scheme, we can obtain three shape functions, as

\[
\begin{align*}
  N_1(x, y) &= 1 - x - y \\
  N_2(x, y) &= x \\
  N_3(x, y) &= y
\end{align*}
\]

Then, the matrix of shape functions can be written as

\[
[N] = \begin{bmatrix}
1 - x - y & 0 & x & 0 & y \\
0 & 1 - x - y & 0 & x & y
\end{bmatrix}
\]

The strain displacement matrix can be obtained from the definition of strain as

\[
\{\varepsilon\} = \begin{bmatrix}
\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \\
\frac{\partial v}{\partial x} & \frac{\partial v}{\partial y}
\end{bmatrix} = \begin{bmatrix}
-1 & 0 & 1 & 0 & 0 \\
0 & -1 & 0 & 0 & 0
\end{bmatrix}
\]

\[
[B] = \begin{bmatrix}
u_1 \\
v_1 \\
u_2 \\
v_2 \\
u_3 \\
v_3
\end{bmatrix}
\]